

Exercise 16

(a) Find the differential dy and (b) evaluate dy for the given values of x and dx .

$$y = \cos \pi x, \quad x = \frac{1}{3}, \quad dx = -0.02$$

Solution

Compute the derivative of y .

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(\cos \pi x) \\ &= (-\sin \pi x) \cdot \frac{d}{dx}(\pi x) \\ &= (-\sin \pi x) \cdot (\pi) \\ &= -\pi \sin \pi x \end{aligned}$$

Consequently, the differential of $y = \cos \pi x$ is

$$dy = -\pi \sin \pi x \, dx.$$

If $x = 1/3$ and $dx = -0.02$, then

$$dy = -\pi \left(\sin \frac{\pi}{3} \right) (-0.02) = -\pi \left(\frac{\sqrt{3}}{2} \right) \left(-\frac{1}{50} \right) = \frac{\pi\sqrt{3}}{100} \approx 0.054414.$$